

Analysing Electric Field Distribution in Non-Ideal Insulation at Direct Current¹

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Abstract –It is not uncommon for young electrical engineers to overlook the influence of the insulation leakage on the electric field distribution at DC or slow-varying voltage. A classic problem is revisited: distribution of the electric field between two infinite parallel plates separated by two layers of isotropic insulation. Systematic mistakes ensuing from incorrect application of the boundary condition that is valid only for electrostatics are analyzed. Instead, a more general boundary condition should be used. It is obtained from the current continuity equation in its integral form and is expressed in terms of the normal components of the density of full current. The equivalent circuit approach is useful as a complementary method to analyzing the problem, especially when the conduction and displacement currents are commensurable. Numerical field solutions are given for two practical insulation systems.

I. INTRODUCTION

Electrical insulation in high-voltage apparatus is stressed by various voltage waveforms. They range from many MHz to 50/60Hz AC voltages and from fast transients to DC voltage. For many devices, slow transients are common. For example, a soft start in a precision power supply may continue several tens of seconds. In the same power supply very fast transients may occur during a load breakdown, e.g., a sparking in an X-ray tube. In long-pulse applications, a DC voltage is established in anywhere from microseconds to many milliseconds, the latter being the case for computer tomography.

Good insulation design calls for the calculation of electric field that depends not only on geometry, property of materials, the voltage amplitude, etc., but on the voltage waveshape as well. For an experienced practicing high voltage engineer, no questions arise in differentiating between electric field distribution at AC, fast and slow transient and steady-state (DC) conditions in identical insulation systems. Of course, in DC systems, the conduction currents govern the field distribution, while during fast transient processes and at AC, presumably at a line frequency of 50Hz and higher, the displacement currents are of the major importance. Put alternatively, the material conductivity is dominant at DC, and the material permittivity is dominant at AC. This is a well-known code of practice [1-3].

Surprisingly, very few of electrical engineering students that had taken regular undergraduate courses on electromagnetic fields identify or associate the electric field problem with insulation conductivity. The same is true with young high voltage engineers and electrical engineers *en masse*. Even more surprisingly, quite a few mature physicists, holding Ph.D. degrees, were perplexed when trying to calculate the distribution of the electric field in a capacitor with layered insulation at DC conditions (see below). This picture observed by the author during many years of professional communication and teaching clearly indicates that there is a gap between the courses on electromagnetic fields and the courses on high voltage engineering, at least on the undergraduate level. On the other side, it is uncommon to offer in these courses a crisp, lucid formulation of the distinction between the electric field distribution in real insulation at steady-state and at AC or time-varying conditions. Accordingly, the purport of this tutorial paper is offering such a formulation; it might save young electrical engineers some pain and embarrassment.

II. CASE STUDY--FLAT CAPACITOR WITH LAYERED INSULATION

Let us revisit a classic problem having basic importance in high voltage engineering--distribution of the electric field between two infinite parallel plates separated by two layers of isotropic insulation (Fig. 1).

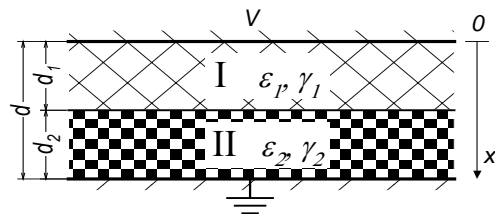


Fig. 1. Flat capacitor with two layers of non-ideal isotropic insulation.

The dielectrics are not ideal, which is reflected by the final values of their conductivities γ_1 , γ_2 . Voltage V that is

¹ This is a revised version of a paper presented at Electrimacs 2008, Quebec, 8-11 June 2008

applied to the plates, is either a constant V_0 (DC case), ramp, or a sinusoidal function of time $V = V_m \sin \omega t$. The examination will be further simplified by adopting $\omega = 2\pi \cdot 50$, and ascribing to the material properties concrete values: $\varepsilon_1 = 2.3\varepsilon_0$, $\varepsilon_2 = 5\varepsilon_0$, where ε_1 , ε_2 are dielectric I, II permittivities, respectively, and ε_0 is the permittivity of free space, $\gamma_1 = 10^{-15} \Omega^{-1} \text{m}^{-1}$, $\gamma_2 = 10^{-12} \Omega^{-1} \text{m}^{-1}$. (Dielectric I may be a polyethylene and dielectric II may be an epoxy compound.) Formation of space charge, temperature, frequency and field dependence of the dielectric properties, etc., are neglected at this stage. Thus, the problem is defined physically. How it is usually approached?

A. Field Analysis

Unfailingly, one recognizes that the problem is described by the Laplace equation in its simplest form:

$$\frac{d^2 \varphi}{dx^2} = 0, \quad (1)$$

where φ is the potential and x the coordinate, as shown in Fig. 1. Integrating (1) in the layers, one readily obtains the following relations:

$$E_1 = \text{const}, E_2 = \text{const}, \quad (2)$$

$$V = E_1 d_1 + E_2 d_2, \quad (3)$$

where E_1 , E_2 are yet unknown electric field components in layers I, II, respectively. A boundary condition is necessary to find the relation between E_1 , E_2 . Here comes a common fallacy. Almost invariably, the boundary condition is written in its simplest, and best-known, form [1, 4, 5, 7, 8]

$$D_{n1} = D_{n2}, \text{ or } \varepsilon_1 E_{n1} = \varepsilon_2 E_{n2}, \quad (4)$$

where D_{n1} , D_{n2} and E_{n1} , E_{n2} are the normal components of the displacement vector and the electric field, respectively. The tangential components are zero in this case in view of symmetry. Equation (3), (4) yield the solution [4, par. 4.3.1]:

$$E_1 = \frac{V}{d_1 + \frac{\varepsilon_1}{\varepsilon_2} d_2}, \quad E_2 = \frac{V}{d_1 \frac{\varepsilon_2}{\varepsilon_1} + d_2}, \quad (5)$$

which is quite acceptable for the adopted values at 50Hz, since the conduction current is negligible compared to the displacement current. Note that this idea usually eludes the students, since they are guided by the boundary condition (4), which utterly discards the conductivity.

However, (4) and (5) are not valid for a DC case (and, in a rigorous approach, never, if the conductivities are not zero), because the surface charge exists on the boundary between the dielectrics. The normal components of the electric field strength are related as follows:

$$\varepsilon_1 E_{n1} = \varepsilon_2 E_{n2} + \sigma, \quad (6)$$

where σ is the surface charge density. Note that (6) is a proper boundary condition for the electrostatic problem only, when σ is prescribed. Otherwise, (6) serves solely for the calculation of σ , after the field distribution has been found [6]. *This point is almost invariably missed.*

A more general boundary condition that is obtained from the current continuity equation in its integral form

$$\oint_A \delta dA = 0 \quad (7)$$

is expressed in terms of the normal components of the current density δ :

$$\delta_{n1} = \delta_{n2}, \quad (8)$$

or

$$\gamma_1 E_{n1} + \varepsilon_1 \frac{\partial E_{n1}}{\partial t} = \gamma_2 E_{n2} + \varepsilon_2 \frac{\partial E_{n2}}{\partial t}. \quad (9)$$

For sine waveforms, (9) transforms to

$$(\gamma_1 + j\varepsilon_1 \omega) E_{n1} = (\gamma_2 + j\varepsilon_2 \omega) E_{n2}. \quad (9a)$$

Note that δ accounts for both the conduction (first member in (9)) and the displacement mechanism (second member in (9)). For most engineering applications, either the conduction current γE is negligible compared to the displacement current $\varepsilon \frac{\partial E}{\partial t}$:

$$\gamma_i E_{ni} \ll \varepsilon_i \frac{\partial E_{ni}}{\partial t}, \quad i=1,2, \quad (10)$$

or the other way around:

$$\gamma_i E_{ni} \gg \varepsilon_i \frac{\partial E_{ni}}{\partial t}, \quad i=1,2, \quad (10a)$$

If the relation (10) holds, (9) transforms to its simplified form (4), and as such is commonly applied to high voltage field problems at 50/60Hz and higher frequencies. For this case study (50Hz), the ratio of the displacement current amplitude to that of the conduction current for layers I, II is $6.39 \cdot 10^6$ and $1.39 \cdot 10^4$, respectively.

In a DC field, the time derivatives are zero; therefore, boundary condition (9) contains only the media conductivities:

$$\gamma_1 E_{n1} = \gamma_2 E_{n2}. \quad (11)$$

Since for the examined problem $\gamma_1 \ll \gamma_2$, the stress in the first layer is much greater than in the second: $E_{n1} \gg E_{n2}$, and if d_1 and d_2 are commensurable, the solution is obtained immediately from (3):

$$E_1 \approx \frac{V}{d_1}, \quad E_2 \approx \frac{\gamma_1}{\gamma_2} E_1 = 0.001 E_1.$$

The exact solution is identical to (5), where the permittivities are substituted by the conductivities:

$$E_1 = \frac{V}{d_1 + \frac{\gamma_1}{\gamma_2} d_2}, \quad E_2 = \frac{V}{d_1 \frac{\gamma_2}{\gamma_1} + d_2}. \quad (12)$$

Equation (11) is a well-known boundary condition that is applied to static field problems in *conducting* media. However, as mentioned already, given the problem Fig. 1, where seemingly insulating materials are shown, students fail to associate it with the proper boundary condition (11). Majority of them were not introduced to a more general boundary condition (9) in preceding courses.

B. Equivalent Circuits

A simpler approach, not involving field quantities, is using equivalent circuits; by obvious reasons, this approach has larger appeal to electrical engineers than to physicists. A quick glance at Fig. 1 readily yields an equivalent circuit Fig. 2, where

$$R_1 = d_1 / \gamma_1, \quad R_2 = d_2 / \gamma_2, \quad C_1 = \epsilon_1 / d_1, \quad C_2 = \epsilon_2 / d_2 \quad (13)$$

are resistances and capacitances of the layers per unit area.

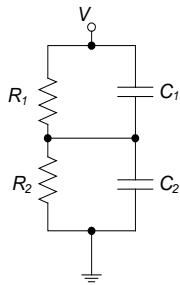


Fig. 2. Equivalent circuit for flat capacitor with two layers of non-ideal insulation.

Solving the corresponding differential equations provides a solution for arbitrary voltage V waveforms allowing to find the voltages across the circuit components, and thus across the insulation layers. Fig. 3a illustrates the voltage distribution across 1-cm-thick layers at the application of a

“long” pulse with a “slow” 30-s ramp, with the materials properties as defined for Fig. 1 (the graphics were conveniently obtained using a PSpice solver); Fig. 3b gives the same except the conductivities are swapped: $\gamma_1 = 10^{-12} \Omega^{-1} \text{m}^{-1}$, $\gamma_2 = 10^{-15} \Omega^{-1} \text{m}^{-1}$. The voltage distributions in Fig. 3a can be assessed, at least at the leading edge, using the frequency sweep of Fig. 3c.

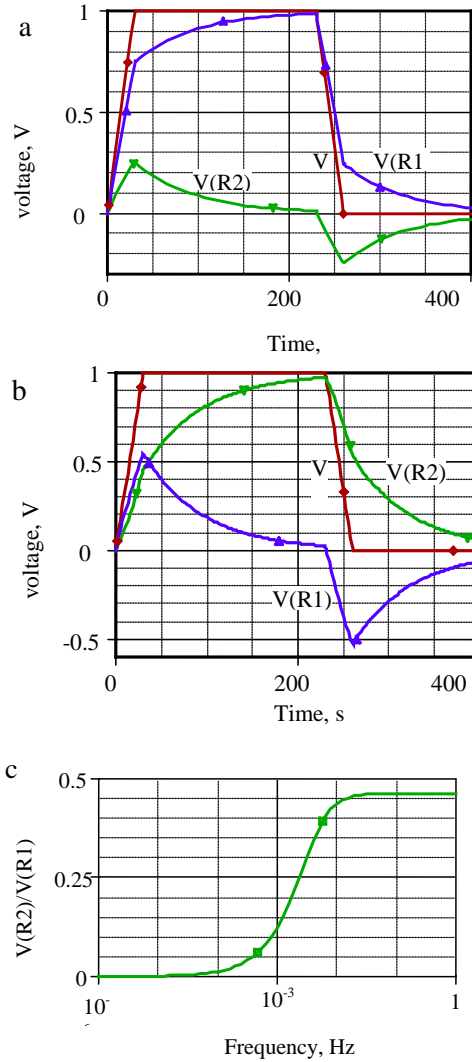


Fig. 3. Solution for equivalent circuit Fig. 2 for parameters as defined for Fig. 1. Layers' thickness $d_1 = d_2 = 0.01$ m, $\epsilon_1 = 2.3 \epsilon_0$, $\epsilon_2 = 5 \epsilon_0$; a - $\gamma_1 = 10^{-15} \Omega^{-1} \text{m}^{-1}$, $\gamma_2 = 10^{-12} \Omega^{-1} \text{m}^{-1}$, b - $\gamma_1 = 10^{-12} \Omega^{-1} \text{m}^{-1}$, $\gamma_2 = 10^{-15} \Omega^{-1} \text{m}^{-1}$, c - $\gamma_1 = 10^{-15} \Omega^{-1} \text{m}^{-1}$, $\gamma_2 = 10^{-12} \Omega^{-1} \text{m}^{-1}$.

As simple as that, the equivalent circuit approach lacks physical insight and should be used as a complementary method in treating field problems in leaky media. In particular, the surface charge formation that is critical to the insulation functioning (it is responsible for the voltage reversal in Fig. 3) is totally hidden behind the circuit equations. Moreover, in more complex geometries calling for a numerical analysis, the circuit approach is quite impotent; it does not suggest a clue to defining the problem. A couple of

such examples are given in the following section.

III. NUMERICAL EXAMPLES

The first example makes use of a coil wound on a high-quality plastic bobbin, e.g., polyethylene ($\epsilon_1=2.3\epsilon_0$), that is further potted in an epoxy ($\epsilon_1=5\epsilon_0$). In the below example, their conductivities are related as 1:100, respectively.

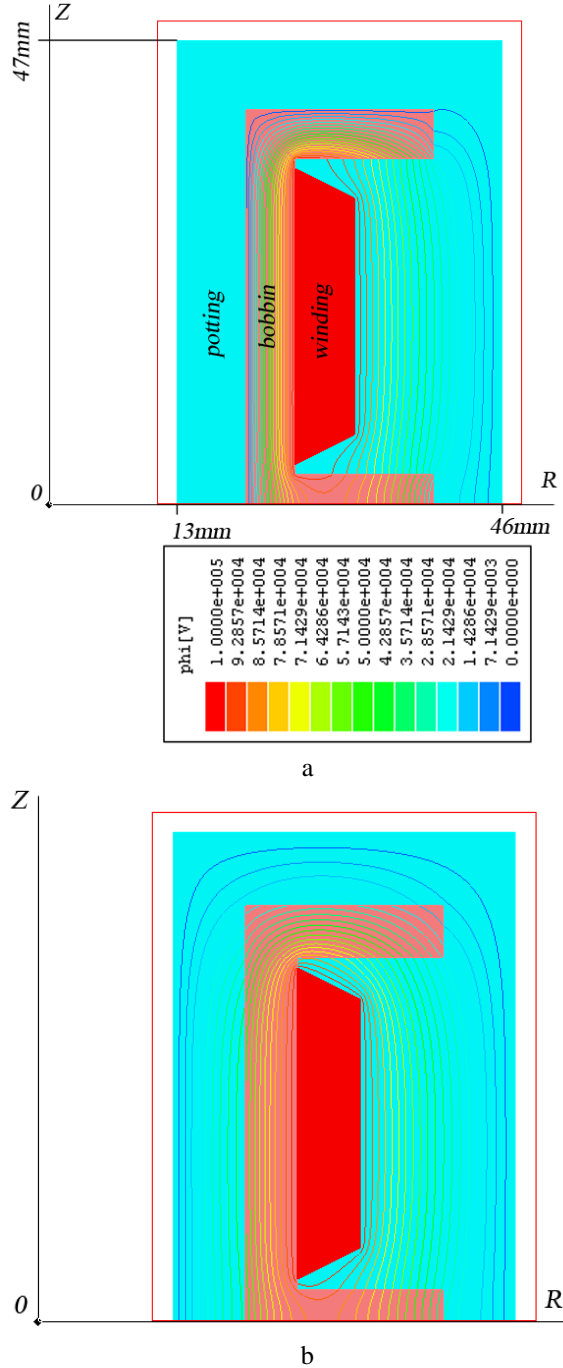


Fig. 4. Distribution of electric field in potted coil. a – DC field (conduction problem); b – AC (electrostatic problem).

Fig.4 shows the field patterns in the form of equipotential lines for DC (a) and AC (transient) cases (b),

respectively. The solution was obtained using Maxwell 2D SV software [9] in an axisymmetric approximation with the mesh size of about 20,000 triangles. Only half of the coil was modeled owing to mirror image symmetry in the R- θ -plane. The outer boundary is maintained at zero potential (except the R- θ -plane, where the normal component of the E -vector is zero), and a voltage of 100kV is applied to the coil. A similar problem was addressed in [10].

For the DC case Fig. 4a, a large difference in the conductivities forces the field to concentrate in the bobbin leaving the potting largely unstressed at the yokes and the inner leg (to the left) of the core. The rationale of this design is relieving the potting material that is more prone to contain defects than its plastic counterpart [10]. On the opposite, the voltage is shared approximately equally by the plastic and potting at AC conditions (Fig. 4b).

The second example depicts the field distribution in an X-ray tube-shield insulation system. Earlier, a similar problem was solved in [11]. With considerable simplification, the problem again was modelled in an axisymmetric approximation with the mesh size of about 20,000 triangles. There are four distinctly different dielectric regions: vacuum inside the tube, the glass envelope, oil, and a plastic barrier.

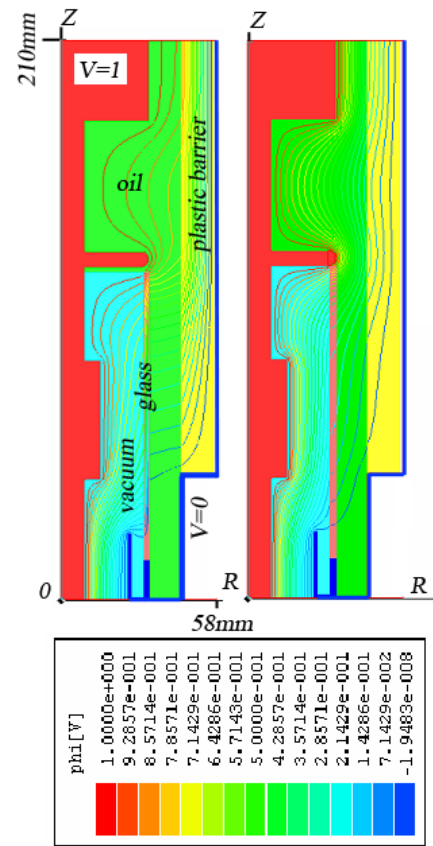


Fig. 3. Distribution of electric field (DC case to the left, AC to the right) in a shielded X-ray tube.

Two cases have been simulated. For the steady-state analysis, the conductivities of materials control the field. In

the following simulations, the conductivities of vacuum, glass, oil and plastic, arguably, were taken in the following ratio: 10^{-13} : 10^{-14} : 10^{-13} : 10^{-12} : 10^{-13} . Their relative permittivities were set as 1, 5.75, 2.25, 3.5, respectively. Note that the vacuum “conductivity” is very strongly field- and polarity dependent [12]. In some actual X-ray tubes, the dark currents increase typically by an order of magnitude for the field increase of 5% [13].

Again, this example illustrates a striking difference between the DC and AC (transient) field distributions. The oil is largely unstressed in the first case, with tendency for even lower stress in the process of the oil aging. The plastic barrier is instrumental for the insulation functioning bearing the brunt of the applied voltage. At AC, the oil is stressed much stronger; contrary to the DC case, the field distribution would remain practically unaffected by time.

Although in the above examples both the geometry and physical properties are treated with great simplification, the field analysis is useful in that it allows a) identifying the basic difference in operation under DC and AC, or transient conditions, and b) finding overstressed regions. An experienced designer may well manage the first part without investing time in detailed simulations using proper boundary conditions and equivalent circuits.

IV. NONLINEAR ASPECTS

After accepting the *existence* of a leakage current-governed field distribution, one starts enquiring about more subtle non-linear aspects. The latter are of the utmost importance at DC or quasi-DC operation. A classic example is a DC power cable under current load. With the central conductor having high temperature, the field becomes stronger at the shield—the situation unthought-of at a line frequency (see, e.g., [3], [14]).

In highly non-uniform fields, space charge effects caused by local ionization in the insulation body, field emission, etc., modify the field distribution considerably. These phenomena are not necessarily limited to the case of partial discharges occurring in the insulation cavities. In fact, a “steady-state” distribution is a misnomer at high stresses under the application of a DC voltage: the space and surface charges form and disappear rendering a dynamic field distribution. Similar phenomena are observed in moving media, e.g., in dielectric liquids under the application of a non-uniform electric field. Even in the *absence* of ionization, polar matter circulates because of the electro-convection. The driving forces that are proportional to the field gradient and the liquid (or gas) dipole moment are quite sufficient to provide effective mixing and cooling in various DC apparatuses, e.g., in oil-insulated power supplies. An example of a “gas pump” resulting in flame extinction is given in [15]. Owing to the movement, hot and cold regions having different conductivities (high and low, respectively) migrate, continuously modifying the electric field distribution. Such behaviour is extremely difficult to quantify, especially in ionized media. We note that although even commercial packages have non-linear solvers allowing modelling

materials properties as a function of field and temperature, calculations of the DC electric field in complex structures seldom carry valuable *quantitative* information. An exception to this statement are the cases when dielectric properties are well-known [16]. However, we believe that even *qualitative* understanding is a valuable tool for successful design.

V. CONCLUSION

The above study shows that the boundary condition (9) provides clear physical basis to typical high voltage problems, where one must account for the insulation non-ideality. On the contrary, the boundary condition (4) is misleading in that it does not contain the insulation conductivity; it should be introduced as a reduction of (9). Equation (6) does account for the conduction current but has no use for the electric field calculation in real-life insulation.

More complicated cases, when the conduction and displacement currents are commensurable (for the examined problem of section II, it is a subherz range), should be treated more rigorously. Likewise, attention should be paid to non-linear issues. Note that for simple insulation systems, such as multilayer flat, cylindrical or spherical capacitor, the problem is handled conveniently by using equivalent R-C circuits. This approach works well with electrical engineering students.

VI. ACKNOWLEDGMENT

The author acknowledges the support given by Spellman High Voltage Electronics Corporation in making this work published.

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